Blood Flow Modelling in Compliant Arteries

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Abstract

In this paper we introduce some basic differential models for the description of blood flow. The focus of the work is on modeling blood flow in medium-to-large systemic arteries. We study the flow of an incompressible viscous Newtonian fluid through a tube with compliant walls. The tube is assumed to be straight, long, with circular cross-sections of variable radius. Elastic properties of the tube wall are described by a linear membrane shell equations.

The Navier-Stokes equations for an incompressible viscous fluid are used to model the flow. Applying asymptotic techniques, we derive a set of useful equation that used to obtain the final model of the pressure and by using Bessel's function velocity. We comment on their mathematical properties, their meaningfulness and accurate numerical simulations.

1. INTRODUCTION

Research in blood flow has a direct impact on our improved understanding and management of human health. In a general sense, blood vessels are hollow utensils for carrying blood. There are three varieties of blood vessels: arteries, veins and capillaries. The network of blood vessels allows the transport of blood to all living cells in the body. Their structure enables the exchange of blood plasma and dissolved molecules between the blood and surrounding arteries [4].The blood flow is a complex phenomenon. We simulate blood flow in compliant arteries. In medium to large vessels such as the human arteries, blood can be modeled as an incompressible Newtonian fluid [3]. The vessel walls we consider to be thin, behaving as a pre-stressed linearly elastic membrane shell [2].

We are interested in a time-dependent flow through the axially symmetric sections of the vascular system. Newtonian fluid through a linearly elastic tube with aspect ratio $\varepsilon = R/L$ (R = radius, L = length of the tube). The coupling between the fluid itself and the boundary is described by requiring the continuity of velocity and the continuity of contact forces applied by the fluid and by the membrane [7], where the forces is coming from the stresses induced by the fluid, and it is in two forms, one is the radial component of the force F_r, and the longitudinal component of the force F_z, here we assume that the lateral wall of the cylinder allows only to the radial displacement, hence the longitudinal displacement is negligible.

This presentation is organized as follows. In section 2, we construct the basic equations of the problem. In Section 3, we present the description of rescaled problem and Asymptotic expansions I, II. In section 4, we describe detail of rewriting the problem using Asymptotic analysis and Bessel's function for model deriving and then we obtain the velocity and the pressure of blood in the arteries which leads to reduced equation of pressure.

2. Blood Flow Modelling

To describe the flow of blood we use the governing equations which consist of the **Continuity** equation and the **Navier-Stokes** equations

$$\nabla \cdot u = 0 \qquad \textit{Continuity}$$
$$\rho\left(\frac{\partial u}{\partial t} + (u \cdot \nabla)u\right) = -\nabla p + \nabla \cdot \left(\mu\left(\nabla u + (\nabla u)^T\right)\right) + F \qquad \textit{Navier - Stokes}$$

where ρ denotes the blood density, $u = (u_1, u_2, u_3)$ is the velocity vector, $F = (F_1, F_2, F_3)$ is the volume force acting on the fluid, P denotes the pressure in the luminal channel, μ is the viscosity of blood, for the newtonian fluid is taken to be a constant, and $\nabla \cdot (\mu(\nabla u + (\nabla u)^T))$ the viscous term.

The term $(u \cdot \nabla)u$ is the nonlinear convection term, and in our case we will consider the fluid of high viscosity, so the problem is dominated with the viscosity term, and so we can consider the convection term negligible.

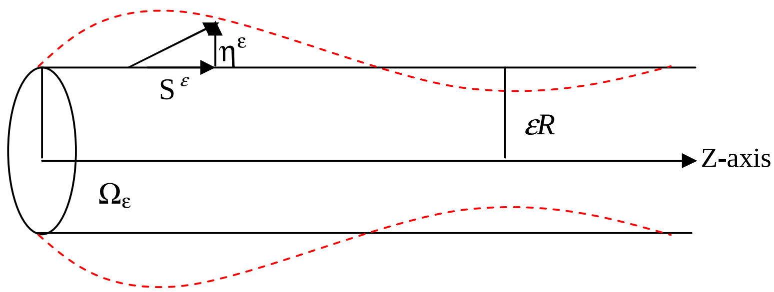

Figure1. The Wall Displacement of the Arteries in Human Body

2.1. Problem Presentation

We study the flow of an incompressible, viscous Newtonian fluid through a cylinder (Figure1) with compliant walls. In the reference state the cylinder is $L > 0$ units long, and $2r > 0$ units wide. The aspect ratio $\varepsilon = r/L > 0$ is assumed to be small, and for each ε introduce Ω_ε to be

$$\Omega_\varepsilon = \left\{x \in R^3; x = (r\cos\theta, r\sin\theta, z), r < \varepsilon R + \eta^\varepsilon(z,t), 0 < \theta < 2\pi, 0 < z < L\right\}. \tag{2.1}$$

We study a time dependent flow driven by the time-dependent inlet and outlet boundary data. The compliant cylinder and its boundary deforms as a result of the fluid-structure interaction between the fluid occupying the domain and the cylinder's boundary. Assume that the lateral wall of the cylinder $\Sigma_\varepsilon(t) = \{r = R + \eta^\varepsilon(z,t)\} \times (0, L)$, is elastic [6] and allows only radial displacement. Its motion, described in Lagrangian coordinates, is modeled by the Navier equation for a linearly elastic curved membrane. The radial contact force is given by

$$F_r = -\frac{h(\varepsilon)E(\varepsilon)}{1-\sigma^2}\frac{\eta^\varepsilon}{R^2} + h(\varepsilon)G(\varepsilon)k(\varepsilon)\frac{\partial^2 \eta^\varepsilon}{\partial z^2} - \rho_\omega h(\varepsilon)\frac{\partial^2 \eta^\varepsilon}{\partial t^2} \tag{2.2}$$

where

F_r **:** is the radial component of external forces, $h = h(\varepsilon)$ **:** is the membrane thickness

η^{ε} **:** is the radial displacement from the reference state, ρ_{ω} **:** is the wall volumetric mass

$k = k(\varepsilon)$ **:** is the Timoshenko shear correction factor, $G = G(\varepsilon)$ **:** is the shear modulus

$E = E(\varepsilon)$ **:** is the Young's modulusm, $0 < \sigma < 0.5$ **:** is the Poisson ratio

and $\rho\omega h(\varepsilon)\frac{\partial^2\eta^{\varepsilon}}{\partial t^2}$ is the inertia term that is proportional to the radial acceleration of the vessel wall, $h(\varepsilon)G(\varepsilon)k(\varepsilon)\frac{\partial^2\eta^{\varepsilon}}{\partial z^2}$ is related to the radial pre-stress state of the vessel, and $\frac{h(\varepsilon)E(\varepsilon)}{1-\sigma^2}\frac{\eta^{\varepsilon}}{\varepsilon^2 r^2}$ is the elastic- response function.

Assumptions:

(a) The young's modulus $E(\varepsilon)$, the wall thickness $h(\varepsilon)$, and the shear modulus $G(\varepsilon)k(\varepsilon)$ satisfy the following

$$h(\varepsilon)E(\varepsilon) > \varepsilon \tag{2.3}$$

$$\lim_{\varepsilon \to 0} \frac{h(\varepsilon)E(\varepsilon)}{\varepsilon} = E_0 \in (0,+\infty) \tag{2.4}$$

$$\lim_{\varepsilon \to 0} \varepsilon h(\varepsilon)G(\varepsilon)k(\varepsilon) = G_0 \in [0,+\infty) \tag{2.5}$$

(b) The starting point of the problem the cylinder is filled with a fluid at equilibrium state, this yields to have an initial pressure $P_0 = 0$, and initial velocity $u^{\varepsilon} = 0$ at $t = 0$.

(c) At the equilibrium state only T_z, T_{θ} components of the stress tensor [5] corresponding to the curved membrane $\sum_{\varepsilon}$ are not zero, where $T_z = kG$ and $T_{\theta} = \varepsilon R\Delta P_0$ is the difference between the reference pressure in the tube and the surrounding tissue.

(d) For simplicity assume $\Delta P_0 = 0 \Rightarrow T_{\theta} = 0$

Assume zero angular velocity, in cylindrical coordinates Eulerian formulation of the problem read as:

$$\rho\frac{\partial u_r^{\varepsilon}}{\partial t} - \mu\left(\frac{\partial^2 u_r^{\varepsilon}}{\partial r^2} + \frac{\partial^2 u_r^{\varepsilon}}{\partial z^2} + \frac{1}{r}\frac{\partial u_r^{\varepsilon}}{\partial r} - \frac{u_r^{\varepsilon}}{r^2}\right) + \frac{\partial p^{\varepsilon}}{\partial r} = 0 \quad in \quad \Omega_{\varepsilon} \times R_+ \tag{2.6}$$

$$\rho\frac{\partial u_r^{\varepsilon}}{\partial t} - \mu\left(\frac{\partial^2 u_r^{\varepsilon}}{\partial r^2} + \frac{\partial^2 u_r^{\varepsilon}}{\partial z^2} + \frac{1}{r}\frac{\partial u_z^{\varepsilon}}{\partial r}\right) + \frac{\partial p^{\varepsilon}}{\partial z} = 0 \quad in \quad \Omega_{\varepsilon} \times R_+ \tag{2.7}$$

$$\frac{\partial u_r^{\varepsilon}}{\partial r} + \frac{\partial u_z^{\varepsilon}}{\partial z} + \frac{u_r^{\varepsilon}}{r} = 0 \quad in \quad \Omega_{\varepsilon} \times R_+ . \tag{2.8}$$

The coupling between the fluid and structure is obtained through the kinematic condition requiring continuity of the velocity evaluated at the deformed interface $\sum_{\varepsilon}(t)$

$$u_r^{\varepsilon} = \frac{\partial \eta^{\varepsilon}}{\partial t} \quad \Sigma_{\varepsilon} \times R_+ \tag{2.9}$$

$$u_z^{\varepsilon} = 0 \quad \Sigma_{\varepsilon} \times R_+ . \tag{2.10}$$

Equation (2.10) is coming from the continuity of a long the longitudinal displacement S^{ε}, and since we assume $S^{\varepsilon} = 0$ then $\frac{\partial s^{\varepsilon}}{\partial t} = 0$.

We set the radial force F_r in equation (2.2) equal to the radial displacement component of the stress exerted by the fluid to the membrane

$$-F_r = (p^{\varepsilon} I - 2\mu D(u^{\varepsilon}))\vec{e}_r . \vec{e}_r = p^{\varepsilon} - 2\frac{\partial u_r^{\varepsilon}}{\partial r} \quad on \quad \Sigma_{\varepsilon} \times R_+ \tag{2.11}$$

$$D(\psi) = \begin{pmatrix} \frac{\partial \psi_r}{\partial r} & 0 & \frac{1}{2}(\frac{\partial \psi_r}{\partial z} + \frac{\partial \psi_z}{\partial r}) \\ 0 & \frac{\psi_r}{r} & 0 \\ \frac{1}{2}(\frac{\partial \psi_r}{\partial z} + \frac{\partial \psi_z}{\partial r}) & 0 & \frac{\partial \psi_z}{\partial z} \end{pmatrix}$$

where $D(u^{\varepsilon})$ is the summarized gradient of the velocity given by

$$D(u^{\varepsilon}) = \frac{1}{2}(\nabla u^{\varepsilon} + (\nabla u^{\varepsilon})^T). \tag{2.12}$$

Physically, it is reasonable to consider the surface is stable at the starting point, the initial velocity is zero

$$\eta^{\varepsilon} = \frac{\partial \eta^{\varepsilon}}{\partial t} = 0 \ on \ \Sigma_{\varepsilon} \times \{0\}. \tag{2.13}$$

The boundary data of the system is given by

$$u_r^{\varepsilon} = 0 \ \ and \ \ p^{\varepsilon} = 0 \ \ on \ \ (\partial\Omega_{\varepsilon} \cap \{z = 0\}) \times R_+ \tag{2.14}$$

$$u_r^{\varepsilon} = 0 \ \ and \ \ p^{\varepsilon} = A(t) \ \ on \ \ (\partial\Omega_{\varepsilon} \cap \{z = L\}) \times R_+ \tag{2.15}$$

$$\eta^{\varepsilon} = 0 \ \ for \ \ z = 0, L \ \ \forall t \in R_+. \tag{2.16}$$

3. Rescaled Problem and Asymptotic Expansion

In this section we will present the problem in it's rescaled form, using a non-dimensional approach.

3.1. Problem in Rescaled Form

To obtain the reduced equation as a rescaled problem we write the equation in non-dimensional form, this should have to be done in order to deal with problem as an easy mathematical model, and this will be done by introduction a new variables. Introduce the non-dimensional independent variables $\tilde{r} = \varepsilon r$ and $\tilde{t} = \frac{\varepsilon^2}{\mu} t$, hence

$$\Omega_{\varepsilon} \to \Omega_1$$

$$u^{\varepsilon} \to u(\varepsilon)$$

$$u^{\varepsilon} = u_r^{\varepsilon}\vec{e}_r + u_z^{\varepsilon}\vec{e}_z \to u(\varepsilon) = u(\varepsilon)_r \vec{e}_r + u(\varepsilon)_z \vec{e}_z .$$

Substitute the new values in Navier-Stoke equation (2.6), equation (2.7) and equation (2.8) to get the following:

$$\frac{\mu}{\varepsilon^2}\frac{\partial u(\varepsilon)_r}{\partial t} - \mu\left(\frac{1}{\varepsilon^2}\frac{\partial^2 u(\varepsilon)_r}{\partial r^2} + \frac{\partial^2 u(\varepsilon)_r}{\partial z^2} + \frac{1}{\varepsilon^2}\frac{1}{r}\frac{\partial u(\varepsilon)_r}{\partial r} - \frac{1}{\varepsilon^2}\frac{u(\varepsilon)_r}{r^2}\right) + \frac{1}{\varepsilon}\frac{\partial p(\varepsilon)}{\partial r} = 0 \tag{3.1}$$

$$\frac{\mu}{\varepsilon^2}\frac{\partial u(\varepsilon)_z}{\partial t} - \mu\left(\frac{1}{\varepsilon^2}\frac{\partial^2 u(\varepsilon)_z}{\partial r^2} + \frac{\partial^2 u(\varepsilon)_z}{\partial z^2} + \frac{1}{\varepsilon^2}\frac{1}{r}\frac{\partial u(\varepsilon)_z}{\partial r}\right) + \frac{\partial p(\varepsilon)}{\partial z} = 0 \tag{3.2}$$

$$\frac{1}{\varepsilon}\frac{\partial u(\varepsilon)_r}{\partial r} + \frac{\partial u(\varepsilon)_z}{\partial z} + \frac{1}{\varepsilon}\frac{u(\varepsilon)_r}{r} = 0 . \tag{3.3}$$

According to the new values in Navier-Stoke equation (2.6), equation (2.7) and equation (2.8) to get the following:

$$F_r = -\frac{h(\varepsilon)E(\varepsilon)}{1-\sigma^2}\left(\frac{\sigma}{\varepsilon R}\frac{\partial s^\varepsilon}{\partial z} + \frac{\eta^\varepsilon}{\varepsilon^2 R^2}\right) + h(\varepsilon)G(\varepsilon)k(\varepsilon)\frac{\partial^2\eta^\varepsilon}{\partial z^2} - \rho_\omega h(\varepsilon)\frac{\varepsilon^2}{\mu^2}\frac{\partial^2\eta^\varepsilon}{\partial t^2}. \qquad (3.4)$$

In the sense of the continuity equations and the lateral boundary conditions we get the following

$$u(\varepsilon)_r = \frac{\varepsilon^2}{\mu}\frac{\partial\eta^\varepsilon}{\partial t} \quad on \quad \Sigma_\varepsilon \times R_+ \qquad (3.5)$$

$$u(\varepsilon)_z = 0 \quad on \quad \Sigma_\varepsilon \times R_+ \qquad (3.6)$$

$$-F_r = p(\varepsilon) - 2\mu\frac{1}{\varepsilon}\frac{\partial u(\varepsilon)_r}{\partial r} \quad on \quad \Sigma_\varepsilon \times R_+ \qquad (3.7)$$

where $D_\varepsilon(\nu(\varepsilon)) = \frac{1}{2}\left(\nabla u(\varepsilon) + (\nabla u(\varepsilon))^T\right)$.i.e. if $\varphi = \varphi_r \vec{e}_r + \varphi_z \vec{e}_z$,

$$D_\varepsilon(\varphi) = \begin{pmatrix} \frac{1}{\varepsilon}\frac{\partial\varphi_r}{\partial r} & 0 & \frac{1}{2}\left(\frac{\partial\varphi_r}{\partial z} + \frac{1}{\varepsilon}\frac{\partial\varphi_z}{\partial r}\right) \\ 0 & \frac{\psi_r}{r} & 0 \\ \frac{1}{2}\left(\frac{\partial\varphi_r}{\partial z} + \frac{1}{\varepsilon}\frac{\partial\varphi_z}{\partial r}\right) & 0 & \frac{\partial\varphi_z}{\partial z} \end{pmatrix}$$

The initial conditions read:

$$\eta^\varepsilon = \frac{\partial\eta^\varepsilon}{\partial t} = 0 \quad on \quad \Sigma_\varepsilon \times \{0\}. \qquad (3.8)$$

The boundary data:

$$u(\varepsilon)_r = 0 \;\; and \;\; p(\varepsilon) = 0 \;\; on \;\; (\partial\Omega \cap \{z=0\}) \times R_+,$$
$$u(\varepsilon)_r = 0 \;\; and \;\; p(\varepsilon) = A(t) \;\; on \;\; (\partial\Omega \cap \{z=L\}) \times R_+,$$
$$\eta^\varepsilon = 0 \;\; for \;\; z = 0, \quad \eta^\varepsilon = 0 \;\; for \;\; z = L \;\; and \;\; \forall t \in R_+.$$

3.2. Asymptotic Expansions

Since now we have uniform estimates for $(u(\varepsilon)_r, u(\varepsilon)_z, p(\varepsilon), \eta^\varepsilon)$, which are valid for their time derivatives as well, here we call this vector X^ε ($X^\varepsilon = (u(\varepsilon)_r, u(\varepsilon)_z, p(\varepsilon), \eta^\varepsilon)$).
We can define the correct asymptotic expansions for X^ε.

3.2.1. Asymptotic Expansions I

$$u(\varepsilon)(z,r,t) = \frac{\varepsilon^2}{\mu}\sum_{i\geq 0}\varepsilon^i u^i(z,r,t) \qquad (3.9)$$

$$p(\varepsilon)(z,r,t)\sum_{i\geq 0}\varepsilon^i p^i(z,r,t) \qquad (3.10)$$

$$\eta^\varepsilon(z,t) = \varepsilon\sum_{i\geq o}\varepsilon^i\eta^i(z,t) \qquad (3.11)$$

3.2.2. Asymptotic Expansion II

$$u(\varepsilon)_r(z,r,t) = \frac{\varepsilon^3}{\mu}\sum_{i\geq 0}\varepsilon^i u_r^i(z,r,t) \qquad (3.12)$$

$$u(\varepsilon)_z(z,r,t) = \frac{\varepsilon^2}{\mu}\sum_{i\geq 0}\varepsilon^i u_z^i(z,r,t) \tag{3.13}$$

$$p(\varepsilon)(z,r,t) = \sum_{i\geq 0}\varepsilon^i p^i(z,r,t) \tag{3.14}$$

$$\eta_\varepsilon(z,t) = \varepsilon\sum_{i\geq 0}\varepsilon^i \eta^i(z,t) \tag{3.15}$$

4. Rewriting the Problem Using Asymptotic Expansions

After rewriting the problem in non-dimensional form by introduction the non-dimensional independent variables *r* and *t* and using **expansions I, II** to be the new values of the definite velocity functions $u_r(\varepsilon)$ and $u_z(\varepsilon)$ and the pressure function $p(\varepsilon)$ in the Navier-Stokes equation [2].

4.1. Reduced Problem Using Expansion I

By using asymptotic expansion I

$$u(\varepsilon)(z,r,t) = \frac{\varepsilon^2}{\mu}\sum_{i\geq 0}\varepsilon^i u^i(z,r,t) \tag{4.1}$$

$$p(\varepsilon)(z,r,t) = \sum_{i\geq 0}\varepsilon^i p^i(z,r,t)\,. \tag{4.2}$$

Add this in the above equations we get

$$\frac{\mu}{\varepsilon^2}\left[\rho\frac{\varepsilon^2}{\mu}\left(\frac{\partial u_r^0}{\partial t}+\varepsilon\frac{\partial u_r^1}{\partial t}+\varepsilon^2\frac{\partial u_r^2}{\partial t}+\cdots\right)-\frac{\varepsilon^2}{\mu}\left(\frac{\partial^2 u_r^0}{\partial r^2}+\varepsilon\frac{\partial u_r^1}{\partial r^2}+\varepsilon^2\frac{\partial^2 u_r^2}{\partial r^2}+\cdots\right)\right.$$
$$-\varepsilon^2\cdot\frac{\varepsilon^2}{\mu}\left(\frac{\partial^2 u_r^0}{\partial z^2}+\varepsilon\frac{\partial^2 u_r^1}{\partial z^2}+\varepsilon^2\frac{\partial^2 u_r^2}{\partial z^2}+\cdots\right)-\frac{1}{r}\cdot\frac{\varepsilon^2}{\mu}\left(\frac{\partial u_r^0}{\partial r}+\varepsilon\frac{\partial u_r^1}{\partial r}+\varepsilon^2\frac{\partial u_r^2}{\partial r}+\cdots\right)$$
$$\left.+\frac{1}{r^2}\cdot\frac{\varepsilon^2}{\mu}\left(u_r^0+\varepsilon u_r^1+\varepsilon u_r^2+\cdots\right)\right]+\frac{1}{\varepsilon}\left(\frac{\partial p^0}{\partial r}+\varepsilon\frac{\partial p^1}{\partial r}+\varepsilon^2\frac{\partial p^2}{\partial r}+\cdots\right)=0.$$

From collect the power of ε [1] it is noticed that P_0 is independent of *r*.

$$\frac{\partial u_r^0}{\partial r}+\frac{1}{r}u_r^0=0\Rightarrow\frac{1}{r}\frac{\partial}{\partial r}(ru_r^0)=0\,. \tag{4.3}$$

Substitute $u_r^0 = 0$, this yields also P^1 is independent of *r*. Since $u_r^0 = 0$, this given $\frac{\partial^2 u_r^0}{\partial z^2}=0$.
To simplify the model we introduce of $P = P^0 + \varepsilon P^1$ and $u_r = u_r^1 + \varepsilon u_r^2$, we get

$$\rho\frac{\partial u_r}{\partial t}-\left(\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{1}{r^2}u_r\right)+\frac{\partial p^2}{\partial r}+\frac{\partial p^3}{\partial r}=0 \tag{4.4}$$

with the same notation of *P* and $u_z = u_z^0 + \varepsilon u_z^1$ we can simplify equations

$$\rho\frac{\partial u_z}{\partial t}-\left(\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}\right)+\frac{\partial p}{\partial z}=0. \tag{4.5}$$

From adding equations one can get $$\frac{\partial}{\partial r}(ru_r)+\frac{\partial}{\partial z}(ru_z)=0. \tag{4.6}$$

Now we can summarize the first expansion as: find u_r, u_z and P that satisfy the following equation

$$\begin{cases}\rho\dfrac{\partial u_z}{\partial t}-(\dfrac{\partial^2 u_z}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial u_z}{\partial r})+\dfrac{\partial p}{\partial z}=0\\ \dfrac{\partial}{\partial r}(ru_r)+\dfrac{\partial}{\partial z}(ru_z)=0\end{cases} \quad \text{where} \quad \begin{aligned} p &= p^0+\varepsilon p^1\\ u_r &= u_r^1+\varepsilon u_r^2\\ u_z &= u_z^0+\varepsilon u_z^1.\end{aligned} \tag{4.7}$$

In this way, the same method as in expansion I, here we will use expansion II which is as same as the result of expansion I. And then we again obtain equation (4.7). Let now $\eta=\eta^0+\varepsilon\eta^1$ and insert equation (3.12), equation (3.13) and equation (3.14) into equation (2.2) and equation (3.7) to obtain:

$$F_r=-\frac{h(\varepsilon)E(\varepsilon)}{(1-\sigma^2)\varepsilon R^2}(\eta^0+\varepsilon\eta^1+\cdots)+h(\varepsilon)G(\varepsilon)k(\varepsilon)\varepsilon\frac{\partial^2}{\partial z^2}(\eta^0+\varepsilon\eta^1+\cdots)$$
$$-\rho_\omega h(\varepsilon)\frac{\varepsilon^4}{\mu^2}\varepsilon\frac{\partial^2}{\partial t^2}(\eta^0+\varepsilon\eta^1+\cdots)$$
$$F_r=-(p^0+\varepsilon p^1+\cdots)+2\mu\frac{1}{\varepsilon}\frac{\partial}{\partial r}\left(\frac{\varepsilon^2}{\mu}(\varepsilon u_r^1+\varepsilon^2 u_r^2+\cdots)\right).$$

By using the assumptions, equation (2.4) and equation (2.5) as $\varepsilon\to 0$ with the notations $p=p^0+\varepsilon p^1$ and $\eta=\eta^0+\varepsilon\eta^1$ we get

$$p=\frac{E_0}{R^2(1-\sigma^2)}\eta-G_0\frac{\partial^2\eta}{\partial z^2}+\vartheta(\varepsilon^2). \tag{4.8}$$

By considering $\lambda = 1$, $R = 4.0\times10^{-3}$, $L = 1.0\times10^{-1}$, $E_0 = 14.4\times10^{-1}$ and $\sigma = 5.0\times10^{-1}$, we have $\frac{E_0}{R^2(1-\sigma^2)}=1.2\times10^5$

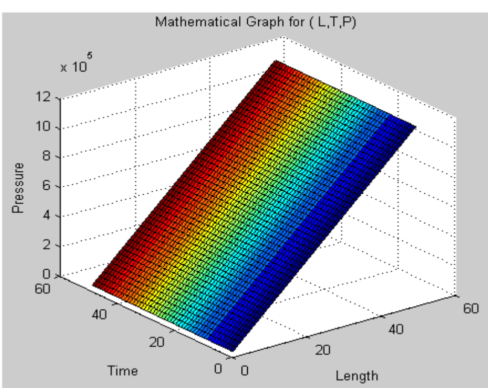

Figure 4.1.Pressure simulation with G_0=0

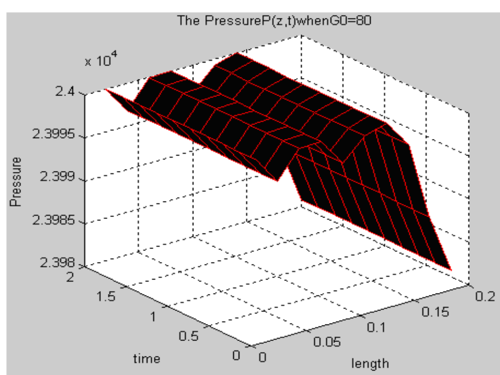

Figure 4.2. Pressure simulation with G_0=80

5. Conclusion

All over the world researcher continue to contribute to the fluid of Computational Biomechanics, which has been gaining increased interest over the last few decades. We have performed simulations of blood flow. However, simulation results can never be more accurate than the applied input data from real patients. With this approach we examine the common geometric variations of the human arteries. We will interest main equation with non-negligible shear modulus G_0 and different functions by using matlab simulation.

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