A SMOOTHING ALGORITHM BASED ON LEAST-SQUARES ESTIMATION FOR GPS DATA PROCESSING

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ABSTRACT: As is well known that noise reduction is of the first importance to single frequency GPS receiver’s data processing. The limiting factors in GPS data processing are accuracy and reliability. In this paper, the theories of Least-Squares (LS) estimation and nonlinear function fitting are applied to solve the problem of data smoothing in the GPS positioning and timing data processing. The paper uses from an actual data collection for evaluating the performance of the algorithm. A simulator is designed and developed for this purpose. The experimental test results and detailed case studies have shown that proposed smoothing algorithm has advantages over the commonly adopted filtering. Quality of the obtained results is good; so that GPS positioning and timing RMS errors reduce to less than 4.2m and 260nsec, respectively.

1. INTRODUCTION

The Global Positioning System (GPS) is a satellite-based navigation system that provides a user with the proper equipment access to useful and accurate positioning-timing information anywhere on the globe. There are a number of error sources inherent in GPS which degrade system accuracy, the major ones behave as 'low-frequency quasi biases', which can be seen as slow drifts in the position solution for a stationary receiver (Cooper and Durrant-Whyte, 1994). These error sources are detailed below (Mosavi, 2006):

Ephemeris Errors: A GPS satellite at an altitude of approximately 20000Km, is subjected to such forces as solar winds and the earth gravitational pull. These forces buffet the satellite, causing the true trajectory to differ from that planned. These forces cannot be estimated and therefore the calculation of satellite position from the ephemeris is generally in error, typically in the region of 15-20 meters.

Propagation Errors: The GPS signal must pass through the earth’s atmosphere, which both delays and refracts electromagnetic waves. The atmospheric levels which have the greatest effect on the signal are the troposphere (sea-level to 50Km) and the ionosphere (20 to 500Km). The ionosphere delays the signal causing an error in the pseudorange of up to 50 meters, the civilian user can only estimate the delay using on ionospheric model. Single frequency GPS receivers use the Klobuchar model which accounts for about half of the delay. The Klobuchar model takes into account such things as sunspot activity and the total electron count, the eight coefficients required being including in the almanac (downloaded by the satellites every 12.5 minutes). The error introduced by the troposphere is impossible to predict in real-time, being affected by the weather conditions between the receiver and satellite (the amount of water in the line of sight to the satellite being the major factor), causing an error in the region of 3-4 meters.

Selection Availability: This is by far the greatest source of positioning error for civilian GPS users, causing errors of up to 50 meters in the psedorange. Selective Availability (SA) is a deliberate error introduced by the US Department of Defense for security reasons to degrade the position solutions of non-military receivers. SA is added to the system in two different ways;

Firstly there is 'Dither' where satellite clocks are deliberately "tweaked" to cause errors in the pseudorange. Secondly, 'Epsilon' is the doctoring of the satellite ephemerides resulting in extra uncertainty of the exact position of the satellites. Used simultaneously these add an uncertainty of between 30 and 50 meters to the measured pseudorange.

Other Sources: There are several other error sources having less effect on system accuracy. The signal is bent by the differing refractive indexes of the atmospheric layers, introducing a maximum error of 0.4m. If the antenna is ill-positioned the signal may be received a number of times as it bounces off reflectors (metal objects) close to the receiver, causing as much as 2 meters error. The satellite clocks become increasingly less accurate with age, leading to synchronization errors with GPS time which increases with age (at present < 1 meter).

For the noise-reduction purpose, smoothing operations are used for reducing noise. There are many existing algorithms of smoothing, but the algorithms with high performance are often expensive to design, implement, as well as compute. On the other hand, the effects of the conventionally, spatial-domain simple algorithms are not very satisfactory (Robert Van Kley P.E., 1994). In engineering applications, curve fitting plays an important role in the analysis and processing of experimental data. In fitting a curve to given data points, there are generally two possible approaches. One is to have the graph of the approximation function pass exactly through the given data points. The method of polynomial approximation of the spline fit utilizes this approach. However, if the data values are experimentally sampled, having the scatter frequently found in such data, this method may yield unsatisfactory results. The second approach, which is usually a more satisfactory one for experimentally sampled-data, uses an approximation function which graphs as a smooth curve having the general shape suggested by the data values, but not usually passing exactly through all of the data points. The second approach is used herein this paper. A Least-Squares (LS) fit is generally a good approach to use with sampled-data. Other criteria besides LS criterion could be employed, such as minimizing the maximum error, but the LS criterion is the most widely used (Zhu et al., 1990).

This paper shows an algorithm to apply the theory of LS estimation and nonlinear function fitting to the data smoothing problem in the field of GPS data processing. Experimental results and detailed comparative studies have shown that the algorithm has more advantages over the commonly adopted filtering. Quality of the obtained results is good. This paper is organized as follows. Section II describes the smoothing algorithm. Experimental results are reported in section III and finally conclusions are presented in section IV.

2. SMOOTHING ALGORITHM

The filtering algorithm is derived as follows. Let it be supposed that sampled-data at the $(2n+1)$ equal-interval points $P_{-n}, P_{-(n-1)}, \ldots, P_{-1}, P_0, P_1, \ldots, P_{(n-1)}, P_n$ are $S_{-n}, S_{-(n-1)}, \ldots, S_{-1}, S_0, S_1, \ldots, S_{(n-1)}, S_n$. T is the sampling interval between every two neighboring sampling points, then by using the following transformation:

$$t_i = \frac{P_i - P_0}{\mathrm{T}} \quad , \quad i = -n, \ldots, +n \tag{1}$$

The above equal-interval sampling points are transformed into $t_{-n} = -n$, $t_{-(n-1)} = -(n-1)$, ... , $t_{-1} = -1$, $t_0 = 0$, $t_1 = 1$, ... , $t_{(n-1)} = (n-1)$, $t_n = n$. Now, consider the following $m-th$ degree polynomial:

$$S(t) = C_0 + C_1 t + \ldots + C_m t^m \tag{2}$$

Its nonlinear characteristic is intended to be used to improve the sampled-data corrupted by the noises. To this end, the coefficients $C_j \; (j = 1, \cdots, m)$ of the above polynomial should be determined in order that the nonlinear curve can sectionally approach to related sampled-data points. By substituting all the $(2n+1)$ (t_i, S_i) points of the sampled-data sequence into equation (2), then it gives the following $(2n+1)$ equations:

$$\begin{cases} (C_0 + C_1 t_{-n} + C_2 t_{-n}^2 + \ldots + C_m t_{-n}^m) - S_{-n} = R_{-n} \\ \qquad\qquad \vdots \qquad\qquad\qquad \vdots \qquad\quad \vdots \\ (C_0 + C_1 t_0 + C_2 t_0^2 + \ldots + C_m t_0^m) - S_0 = R_0 \\ \qquad\qquad \vdots \qquad\qquad\qquad \vdots \qquad \vdots \\ (C_0 + C_1 t_n + C_2 t_n^2 + \ldots + C_m t_n^m) - S_n = R_n \end{cases} \tag{3}$$

Where $R_i \; (i = -n, \cdots, +n)$ is the residual error of approximation or curve fitting. Since a section of the nonlinear curve corresponding to a polynomial can approach but not necessarily pass through every sampled-data point (t_i, S_i), the residues on the right side of those equations in equation (3) will not be all zero. For this reason, an optimization criterion should be selected to determine the coefficients C_j of the polynomial equation (2) which can well sectionally approach the sampled-data points. LS method is an optimization criterion gives an indication of the situation of approximation. In terms of this fitting criterion, the optimal values of the coefficients C_j for the $(2n+1)$ sets of data (t_i, S_i) are the very values which make summation of the squares of residues R_i minimal. Therefore, let:

$$\sum_{i=-n}^{i=+n} R_i^2 = \sum_{i=-n}^{i=+n} [\sum_{j=0}^{j=+m} C_j t_i^j - S_i]^2 = \phi(C_0, C_1, ..., C_m) \tag{4}$$

Which enables the coefficient vector $\phi(C_0, C_1, ..., C_m)$ to be minimized. Thus, the coefficients $C_0, C_1, ..., C_m$ must satisfy the following normal equations:

$$\frac{\partial \phi}{\partial C_k} = 2 \sum_{i=-n}^{i=+n} (\sum_{j=0}^{j=+m} C_j t_i^j - S_i) t_i^k = 0 \quad , \quad k = 0, 1, ..., m \tag{5}$$

or:

$$\sum_{i=-n}^{i=+n} S_i t_i^k = \sum_{j=0}^{j=+m} C_j \sum_{i=-n}^{i=+n} t_i^{k+j} \tag{6}$$

In general, the LS fit of an $m-th$ degree polynomial in equation (2) to $m+1$ data points will of course pass through all the given data points since all the residuals go to zero. However, the polynomial may fluctuate considerably between the data points. To minimize such fluctuations of a polynomial between data points, the degree of the polynomial used should be less than the number of data points used. Although no general rule exists for selecting the most suitable degree, a rule of thumb sometimes used is to select the degree of the polynomial somewhere in the neighborhood of $1/2$ to $3/4$ of number of data points used. The smoothing effect between data points due to using a lower degree polynomial is of course generally accompanied by a curve having a poorer fit to the data points. It should be pointed out that the rule of thumb mentioned above should not be applied indiscriminately in all cases.

If the curve of a third degree polynomial $m=3$ is adopted to fit the five sampled-data points $(2n+1)=5$ before and after, the normal equations would be as follows:

$$\begin{cases} 5C_0+10C_2=S_{-2}+S_{-1}+S_0+S_1+S_2 \\ 10C_1+34C_3=-2S_{-2}-S_{-1}+S_1+2S_2 \\ 10C_0+34C_2=4S_{-2}+S_{-1}+S_1+4S_2 \\ 34C_1+130C_3=-8S_{-2}-S_{-1}+S_1+8S_2 \end{cases} \tag{7}$$

The coefficients C_0, C_1, C_2, C_3 are solved from equation (7). By substituting them into equation (2), and let $t=0$, $t=\pm1$, $t=\pm2$, then the filtering formulas are derived as following:

$$\hat{S}_{-2}=\frac{1}{70}(69S_{-2}+4S_{-1}-6S_0+4S_1-S_2) \tag{8}$$

$$\hat{S}_{-1}=\frac{1}{35}(2S_{-2}+27S_{-1}+12S_0-8S_1+2S_2) \tag{9}$$

$$\hat{S}_0=\frac{1}{35}(-3S_{-2}+12S_{-1}+17S_0+12S_1-3S_2) \tag{10}$$

$$\hat{S}_1=\frac{1}{35}(2S_{-2}-8S_{-1}+12S_0+27S_1+2S_2) \tag{11}$$

$$\hat{S}_2=\frac{1}{70}(-S_{-2}+4S_{-1}-6S_0+4S_1+69S_2) \tag{12}$$

Where $\hat{S}_i$ denotes the filtered result point of S_i. It may be noticed that the coefficients in the above derived formulas have some symmetry. If there are lots of points in a sampled-data sequence to be processed, for the sake of symmetrization, except that the both end points of a data sequence are computed respectively by equations (8), (9), (11), and (12), all the middle points are computed by equation (10). This conesponds to the situation that different 3rd degree LS polynomials are utilized to filter the data in three different sections. It is known from the above presented derivation that for the sampled-data sequence at the equal-interval points, only the values S of several (e.g. 5) points before and after are adopted in the practical filtering formulas, but not the values of the sampling interval T and the positions P_i of those data points in the time sequence. The filtering algorithm of equation (10) actually represents the following convolution window of 1×5 dimension:

$$\frac{1}{35}[-3 \quad 12 \quad 17 \quad 12 \quad -3] \tag{13}$$

With the nonlinear weighting values being symmetrical about center points. With this window operator sliding over the sampled-data sequence from beginning to end, then the filtered result of the data points corresponding to the center point of the filtering window is obtained successively.

Before leaving this section, it might be pointed out that in the most general sense, curve fitting involves the determination of a continuous function which results in the most reasonable or best fit to experimental sampled-data. The particular form of the fitting function will in many cases be known in advance from a consideration of the physical nature associated with the signal being sampled. In other instances, the general appearance of the sampled-data or the particular objective of data processing may suggest the particular form of the fitting function. In general, with a well-chosen approximation function, a LS fit will furnish a good representation of the sampled-data. However, if the approximation function chosen is a poor one, the LS fit will probably be a poor one, and only the best fit for that type of function chosen could be expected.

3. EXPERIMENTAL RESULTS

In order to verify the effectiveness of the smoothing template derived in this paper, a hardware was designed and implemented for real data collection. A simulator was developed by paper author in Microsoft Visual Basic6 for GPS receivers data smoothing. The experimental test setup was implemented and installed on the building of Computer Control and Fuzzy Logic Research Lab in the Iran University of Science and Technology. The observation data received by a low cost and single frequency GPS receiver manufactured by Rockwell Company. Some experimental results are given in the following. The smoothing algorithm parameters selection is based on the experiments. Figure 1 shows $Dx, Dy, and\ Dz$ smoothing for 1000 test data using the method.

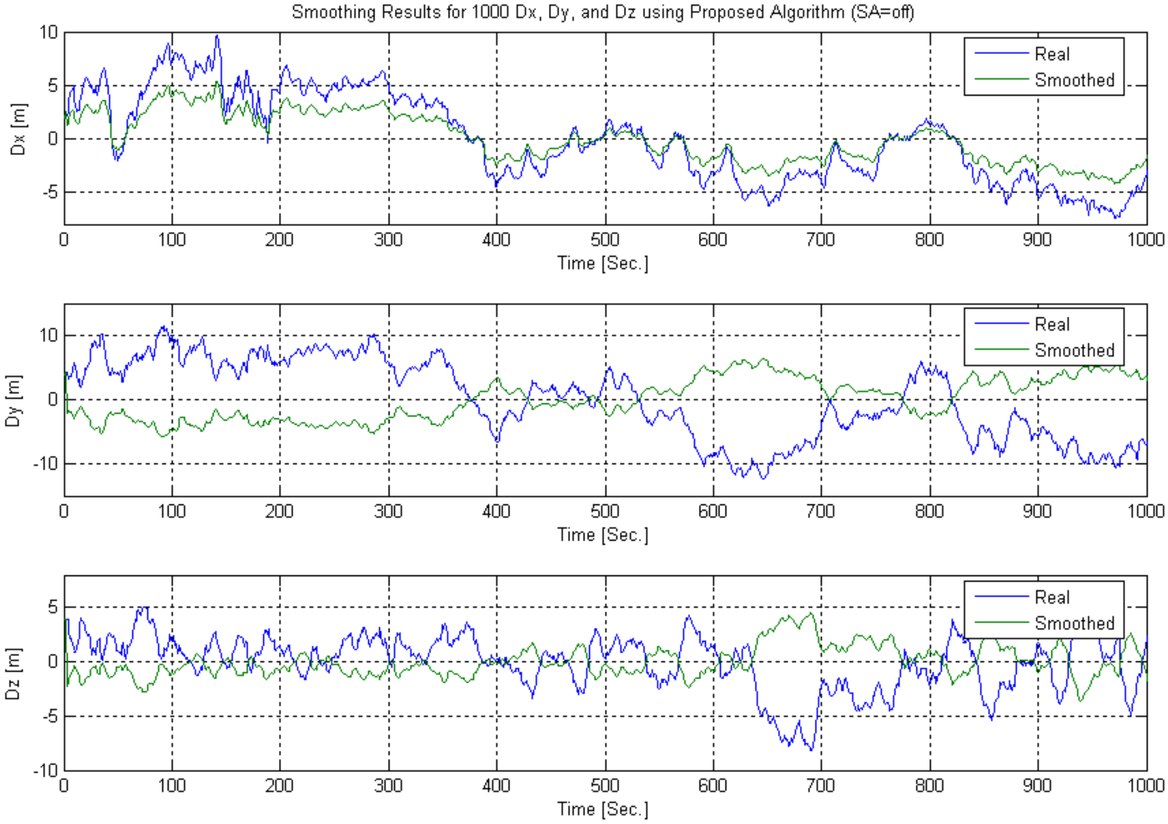

Figure 1: Results of 1000 $Dx, Dy, and\ Dz$ smoothing without SA

In order to evaluate the performance of the algorithm, Root Mean Square (RMS) was used as a measure of closeness between smoothed and observed values (Mosavi, 2007). Tables 1 and 2 show smoothing errors (the difference between the smoothed and real values) statistical significance characteristics for 1000 positioning test data and 500 timing test data, respectively. Tables 1 and 2 show that quality of GPS receiver data noise reduction was good. The proposed template has also less computation than many of the exiting methods.

Table 1: 1000 positioning smoothing errors statistical characteristics without SA

Parameters	**X [m]**	**Y [m]**	**Z [m]**
RMS	2.2827	3.1994	1.4061
Average	-0.0085	0.0036	0.0058
Standard Deviation	0.0721	0.1011	0.0444

Table 2: 500 timing smoothing errors statistical characteristics without SA

Parameters	**t [nsec]**
RMS	259
Average	257
Standard Deviation	12

4. CONCLUSIONS

A GPS data smoothing algorithm based on LS function fitting was presented in the one-dimensional form. The experimental test results on the collected real data are good; so that GPS positioning and timing RMS errors reduce to less than 4.2m and 260nsec, respectively.

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References

Cooper, S., and Durrant-Whyte, H., 1994. A Kalman filter model for GPS navigation of land vehicles. IEEE Conference on Intelligent Robots and System, 1, pp. 157-163.

Mosavi, M. R., 2006. Comparing DGPS corrections prediction using neural network, fuzzy neural network, and Kalman filter. International Journal of GPS Solutions, 10(2), pp. 97-107.

Mosavi, M. R., 2007a. Real time prediction of GPS receivers timing errors using parallel-structure neural networks. International Journal of Geoinformatics. 3(3), pp. 53-61.

Mosavi, M. R., 2007b. Precise real-time positioning with a low cost GPS engine using neural networks. International Journal of Survey Review, 39(306), pp. 316-327.

Mosavi, M. R., 2007c. GPS receivers timing data processing using neural networks: optimal estimation and errors modeling. International Journal of Neural Systems, 17(5), pp. 383-393.

Robert Van Kley P.E., E., 1994. Data smoothing algorithm. IEEE Conference on Position Location and Navigation Symposium, pp. 323-328.

Zhu, H. A., Zheng, N. N., and Zhange, X. N., 1990. A two-dimensional smoothing template for digital image processing. IEEE Conference on Industrial Electronics, 1, pp. 250-255.