Mathematical Base of Spatial Data in Remotely Sensed Image Software and GIS Software

Di Wu[a], Hai Hu[b], Shao-hua Tang[c], Hong-gang Chen[c]
[a]Dept. of Hydrography and Cartography, Dalian Naval Academy, Dalian, China, 116018;
[b]Dept. of Missile and Onboard Cannon, Dalian Naval Academy, Dalian, China, 116018;
[c]Dept. of Operational Commanding, Naval Arms Commanding Academy, Guangzhou, China, 510430
wudiok2468@sina.com.cn; cell phone 138 4088 2486; office phone0086-411-8266 5987

Keywords: Spatial data, Mathematical Base, Conversion, Remotely Sensed Image Processing Software, GIS software

Abstract: The same mathematical base must be given in GIS spatial data and RS image integration for their common Geo-coding. The coordinates systems, such as geographic, projection, geodetic, ellipsoid and datum, all are the mathematical base of spatial data. The selection and transformation of these elements will directly influence the following image processing and geographic information analyses. In order to overcome these disadvantages in RS and GIS softwares, the difference and relation of these coordinates systems were analyzed. And these systems are based on the physical and geometric approximations of the earth's surface.

1. INTRODUCTION

The source of spatial data in GIS and RS software includes terrain maps of various scales, aerial and satellite images, many thematic maps, etc., and these data should be positioned into a common geographical coordinates system or plane coordinates system through coordinate transformation and map projection transformation. The mathematical bases of spatial data include geodesy system, earth elements, map projection and projection parameters, etc., which have the positioning and measuring abilities.

The same mathematical base must be given in GIS spatial data and RS image integration for their common Geo-coding (Dang An-rong, 2000). The mathematical base of spatial data could reduce to two systems, i.e. coordinates system and projection system. But in most cases, the definitions of these two systems are very confused and will influence the following task. Many researchers have given parts of the solution of the integration of coordinates systems, but the particular or specific definitions and analyses haven't been discussed (Hu Peng, 2001; Li Zhen, 2005). In order to overcome these disadvantages in RS and GIS softwares, the differences and relationships of these coordinates systems have been analyzed, which are based on the physical and geometric approximations of the earth's surface.

2.Mathematical base in GIS and RS software

2.1The origin of the problem

A spheroid that approximately substitutes the earth must be found because of the earth's irregular shape in the research of surveying and mapping. Geoid is the datum of surveying because the level axis of the instrument parallels the geoid and the perpendicular axis is vertical to the geoid. The body surrounded by geoid is called ground body, which is a physical approximation of earth's shape. Geoid is the datum of height, called height datum. The local mean sea level is the actual height datum substituting the geoid, such as Huanghai height datum of 1956(Huanghai56) and national height datum of 1985 in China (National85).

But geoid can't be the original plane of projection or the calculation datum of surveying products because it can't be described by mathematical formula, since the undulations of earth's surface and irregular distribution of insight substance have changed the direction of gravity. The geodetic scientists prove through practice that earth's surface approaches an ellipsoid of revolution with minor flattening. To conveniently calculate, the ellipsoid of revolution similar to the size and shape of geoid will be selected, of which minor axis has the same position with the earth's axis and is surrounded with ellipse similar to the earth's meridian. This ellipsoid of revolution is a geometric approximation of earth's shape. The surface of the ellipsoid can be expressed by mathematical formula, which can be easily used in the area of surveying and mapping.

Each country has adopted a datum that fits its own country well so as to conveniently describe the ellipsoid. The fitting process is to describe the irregular earth as ellipsoid or spheroid by establishing geodetic coordinates system (also called geographic coordinates system), including the determination of the earth's shape and size, the orientation and direction of the spheroid and the decision of the origin of the earth. Thus, different ellipsoid elements, different orientation and direction will produce different geodetic coordinates in different countries or in different area of a same country. Because most GIS and RS software are based on plane table system, the projection method must be used to transform three dimensional ellipsoid to two dimensional map.

2.2 Anatomy of each system

Geographic coordinate system and Projection coordinate system is the two expressing forms of mathematical base in RS and GIS softwares. Take MapInfo for example, Table.1 and Table.2 shows the relationships and differences of the two coordinates systems.

English parameters are given in general international software, but the translations are not absolutely exact. For example, spheroid and ellipsoid in GIS software are with the same meaning of ellipsoid of revolution, but ellipsoid is not the commonly used term. Datum is another example in the table, which is always translated into datum plane, but actually it represents geographical coordinates, also geodetic coordinates.

3. Annotation of coordinates systems and analyses of their correlations

3.1 Annotation of coordinates systems

3.1.1Geodetic coordinates

With the demanding of surveying and mapping precision, different countries have adopted different geodetic coordinates systems. Even the same country have existed different geodetic coordinates systems at different period. As China have administrated land and ocean separately for a relatively long time, there are both Beijing 54 and Xi'an 80 coordinates systems. Large or middle cities in China, especially coastal cities, have been adopted their independent or relatively independent coordinates systems so as to meet the rapid development of city construct, these coordinates systems are called local coordinates systems (Su Xu-ming, 2001).

3.1.2 Geocentric coordinates

The definition of geocentric coordinates system includes three aspects: the superposition of the ellipsoidal center and the earth's centroid, also the superposition of the minor axis and the earth's axis and the geodetic prime meridian paralleling to the chronometer meridian. Geocentric coordinates system has no ground origin point, while the position of every point is calculated according to geocentric point. So the relative position error among ground points is independent of the distance. This coordinates system is widely used in satellite positioning and spatial technology (see Table.3).

3.1.3 Geographic coordinates

When the point is on the reference ellipsoid, the geographic coordinates system will be adopted, the point of which is expressed with longitude and latitude. Every point on the ellipsoid will be expressed by geographic coordinate (B, L), which is the geographic situation composing with longitude and latitude. If the point isn't on the ground, the position of it will add the geodetic height H. Geographic coordinate (B, L) are also called geodetic coordinates, and it is the most basic coordinate in geodetic surveying, projection calculation, navigation and aviation positioning, and also basic coordinate in the researching of earth's shape, satellite navigation and spaceflight. In RS and GIS software, the geographic coordinate is the widely used coordinate. Then the content of Table.1 will be easily understood.

3.1.4 Projection coordinates

Projection coordinates system is a plane coordinates system based on certain geodetic coordinates system. That is what the meaning of content in Table.2: every projection coordinates system must have the parameters of geographic coordinates. As a plane coordinates system, the projection coordinates system is expressed by plane rectangular coordinate (X, Y).

3.1.5 Vertical datum

The terrain elevation and ocean depth of spatial data have different vertical coordinate references, including elevation and depth datum. In order to distinguish with plane coordinates system, the elevation, depth datum and united height and depth datum are generally called vertical datum. When using terrain maps and charts in different decade and place, the problems of different elevation, depth datum and different height/depth datum always exist.

3.2 Analyses of correlations among coordinates systems

3.2.1 Transformation among coordinates systems

The relation between ellipsoid and geodetic coordinates system is one versus many. Because geodetic coordinates system is built up on the ellipsoid, while ellipsoid can’t represent geodetic coordinates system and one ellipsoid will definite various different geodetic coordinates systems. For example, Pulkovo1942 coordinates system of previous Soviet Union and Afgooye coordinates system of African Somali are both adopting ellipsoid of Krasovsky, but their geodetic coordinates systems are totally different.

Geodetic coordinates system is actually geographic coordinates system. The two names are respectively used in surveying and mapping. Coordinates system is the norm of spatial data, and it is also the base of GIS. There are three elements in three dimension spatial coordinates, including the original point of coordinates system, the direction of coordinate axis and the scale of coordinates unit. The earth ellipsoid must be introduced so as to express the spatial data with commonly used geodetic coordinates (including longitude, latitude and height). Furthermore, projection method must be introduced to express geodetic coordinate (longitude and latitude) with plane coordinate. Thus three dimension space coordinate, geographic coordinate and projection coordinate are three different forms of coordinates systems, which have rigorous mapping relationship (Parker, B., K. Hess, 2003; Wang Jie-xian, 2003). The conversion of coordinates with different datums needs 7 parameters, i.e. three displacement parameters, three rotation parameters and one scale parameter.

A datum conversion is the process of bringing coordinate values referenced to one defined datum into another datum system. Complete datum conversion is based on seven parameter transformations that include three translation parameters, three rotation parameters and a sacle parameter. The coordinates conversion formula is as the following:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix} + \begin{bmatrix} D_S & R_Z & -R_Y \\ -R_Z & D_S & R_X \\ R_Y & -R_X & D_S \end{bmatrix} \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} \tag{1}$$

Therein, X, Y, Z and X', Y', Z' are two groups of coordinates ; T_X, T_Y, T_Z is referred to translation parameters ; R_X, R_Y, R_Z expresses rotation parameters ; D_S is the scale parameter.

The values of conversion parameters in different coordinates can be obtained in their official files. The transformation between two different coordinates systems should add two other conversion parameters, i.e. the corresponding parameters of ellipsoid ($\mathrm{d}a$, $\mathrm{d}\alpha$). The conversion formula is called differential coefficient formula of geographic coordinates or conversion ellipsoid differential coefficient formula.

3.2.2 Transformation among vertical datums

The relations among different vertical datums sometimes refer to the height conversion between two zero surfaces or sometimes refers to the elevation conversion with common points due to the need of tasks. But whatever the relations are, the influencing factors come from many aspects. Besides the crustal vertical movement, there are some other factors that can be concluded as the following:

Different origin data;
Different observation value;
Different scope of observation value;
Different processing (including correction value and adjustment);

Take Huanghai vertical datum of 1956 for example, various local values of vertical datums subtract from Huanghai56, the results are as the following: the average difference between Dalian vertical datum and Huanghai56 is -0.027m; Dagu is +1.217m; Deposed Huanghe is +0.130m; Wusong is +1.630; Kanmen is –0.228m or –0.237m; Zhujiang is from –0.370m to –0.720m.

The elevation of Qingdao level origin according to Huanghai56 is 72.289m, but according to National85, it is 72.260m. Thus when compiling and charting between data of these two vertical datums, the correction will necessarily be solved as the following formula:

$$H1=H2+\Delta h \qquad (2)$$

Thereinto: H1--Elevation of Huanghai56 vertical datum;
H2--Elevation of National85 vertical datum;
Δh--correction coefficient （the value is 0.029m）.

4.Key questions in selection and transformation of coordinates system of RS and GIS software

4.1Definition and transformation of geographic coordinates system

In the software of RS and GIS, the transformation of two Datums can be realized through 7 parameters method. For example, the geographic coordinates system can be defined through translation between local geodetic coordinate system and WGS-84. Suppose Xg, Yg, Zg as three axises of geocentric coordinates system of WGS-84, At, Yt, Zt as three coordinates axises of local geodetic coordinates system. Then the 7 parameters that define the local datum will be expressed respectively by three displacement parameters ΔX, ΔY, ΔZ, which express the value of displacement of coordinate origin, and by three rotation parameters εx, εy, εz, which express the rotation angles circling three axises Xt, Yt, Zt, and finally by a scale parameter K, which is a revised parameter adjusting the size of ellipsoid.

The definition of geodetic coordinates system in MapX is as following:

Datum.Set（Ellipsoid, ShiftX, ShiftY, ShiftZ, RotateX, RotateY, RotateZ, ScaleAdjust, PrimeMeridian）

In practice, if enough control points are known in the working area of both Beijing 54 and WGS-84 coordinates system, the 7 parameters or three parameters will be directly calculated. If the working area has three known Beijing 54 and WGS-84 coordinates control points, the following formula will be used to calculate the parameters（A、B、C、D、E、F）:

$$x_{54} = Ax_{84} + By_{84} + C, \quad y_{54} = Dx_{84} + Ey_{84} + F \qquad (3)$$

And a surplus point will be used to check up. If only one control point is known, the difference between coordinates of Beijing54 and WGS-84 could be regarded as displacement parameter, and the precision will be under a certain degree in relatively small working area.

4.2 Definition and transformation of projection coordinates system

Among the national basic scale map series in China (1: 10 000, 1:25 000, 1:50 000, 1:100 000, 1:250 000, 1:500 000, 1:1 000 000), Gauss-Krüger projection is adopted in the greater than or equal to 1:500 000 scale maps, normal conformal conical projection is adopted in less than 1:500 000 scale maps, i.e. Lambert projection with two standard parallels, normal conformal secant cylindrical projection (Mercator projection) is adoped in less than 1:500 000 scale charts. Thus the projections corresponding to national basic scale map series must be adopted in RS and GIS softwares in China(Li Min, Shen Yun-Zhong, 2004).

The definition of coordinates system in MapX includes two parts of parameters: parameters of geographic coordinates system and of projection system. The concrete methods are as the following:

CoordSys.Set

(Type,[Datum],[Units],[OriginLongitude],[OriginLatitude],[StandardParallelOne],[StandardParallelTwo],[Azimuth],[ScaleFactor],[FalseEasting],[FalseNorthing],[Range],[Bounds],[AffineTransform]);

The parameter series of projection system of Gauss-Krüger projection, Lambert projection and Mercator projection are as the following:

Gauss-Krüger projection: Projection code (Type), geodetic coordinates system (Datum), Unit, centre longitude (OriginLongitude), original latitude (OriginLatitude), Scale Factor, coordinates offsets toward east and north.

Lambert projection: Projection code (Type), geodetic coordinates system (Datum), Unit, centre longitude (OriginLongitude), original latitude (OriginLatitude), Standard Parallel One, Standard Parallel Two, coordinates offsets toward east and north.

Mercator projection: Projection code (Type), geodetic coordinates system (Datum), Unit, centre longitude (OriginLongitude), original latitude (OriginLatitude), Standard Parallel.

5. Concluding remarks

The relatively thorough and systematical explanations of the conceptions are presented while using geographic and projection coordinates systems in the international RS and GIS softwares. As there are some certain reasons, the problems of disunion of mathematical base of spatial data in various countries will exist for a long time. One of the problems is lacking the traditional Chinese Beijing 54 and Xi’an 80 datums, and in this study, according to various precision demand, the practical transformation methods are brought forward based on geographic coordinates system and projection system; furthermore, the concrete definitions and parameters of three projections are presented according to the difference between relief map and chart in China. All these works will be the premise of the following remotely sensed image processing and geographic information analyses.

6. ACknowledgements

National Natural Science Fund supported the study in this paper, “Research on Remote Sensing Monitoring and the Unification of Sea Land Datum in Coastal Zone”(No. 40371097).

References

Dang An-rong and Wang Xiao-dong. (2000). ERDAS IMAGING remote sensed processing method: human and environmental research centre of TsingHua University.

Hu Peng. (2001). Study of spatial mathematical base of big GIS and digital earth information science edition of Wuhan University Journal, 296-301.

Li Zhen, Gao Yu. (2005). The batch transformation of Gauss-Krüger projection based on ARC/INFO. Forest industry investigation and programming, 15-17.

Su Xu-ming. (2001). Add the self-defined map projection in MapInfo software, Beijing surveying and mapping, 19-21.

Li Min, Shen Yun-zhong. (2004). The definition and transformation of electronic map based on MapInfo, Surveying and Mapping engineering, 28-31.
Parker, B., K. Hess, D. Milbert and S. Gill (2003). A National Vertical Datum Transformation Tool, Sea Technology .44(9), 10-15.
Wang Jie-Xian, Wang Jun, Lu Cai-ping (2003). Transformation question of WGS-84 and Beijing 54 coordinates system, Journal of geodesy and geodynamics.
Hu Ming-cheng, Lu Fu. (1993). Modern geodesy, Beijing surveying and mapping press.

Table.1. Geographic coordinates system

Angular Unit: Degree (0.017453292519943299)
Prime Meridian: Greenwich (0.000000000000000000)
Datum: D_Beijing_1954
Spheroid: Krasovsky_1940
Semimajor Axis: 6378245.000000000000000000
Semiminor Axis: 6356863.018773047300000000
Inverse Flattening: 298.300000000000010000

Table.2. Projection coordinates system

Projection: Gauss_Krüger
False_Easting: 500000.000000
False_Northing: 0.000000
Central_Meridian: 117.000000
Scale_Factor: 1.000000
Latitude_Of_Origin: 0.000000
Linear Unit: Meter (1.000000)
Geographic Coordinate System:
Angular Unit: Degree (0.017453292519943299)
Prime Meridian: Greenwich (0.000000000000000000)
Datum: D_Beijing_1954
Spheroid: Krasovsky_1940
Semimajor Axis: 6378245.000000000000000000
Semiminor Axis: 6356863.018773047300000000
Inverse Flattening: 298.300000000000010000

Table.3. Common used ellipsoid parameter

Ellipsoid	Age	MapInfo Code	Semimajor	Semiminor	1/flattenning
Krasovsky	1940	3	6378245	6356863	298.3
IAG75	1975	31	6378140	6356755	298.25722101
WGS-84	1984	28	6378137.000	6356752.314	298.257223563