The Vector of Wavelet –Fractal and Application in the satellite remote sensing

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Abstract: According to Wavelet analytical principles and Fractal theories, we put forward a new physical quantity: the Vector of Wavelet –Fractal. We built up a new kind of method that analyses and calculates the nonlinear time series. Applying the ways and means we analysis rain-storm's nonlinear characteristic in the satellite remote sensing information, its result fitted with the variety tendency of actual rainfall in 1998, Wu Han.
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1. Introduction

Along with the development of science, it is understood expressly that nature actually presents all nonlinear property in the interaction of various quantities. As people usually have no united method to follow for a long time [1]. This restricts the development of Natural Science, including Atmospheric Sciences. For example, the valley of Yangtze River, She River, Songhua River took place strong rainfall in 1998. It couldn't accurately forecast if making use of the normal regular way to forecast. In this text, according to Wavelet analytical principles and Fractal theories, we put forward a new physics quantity: the Vector of Wavelet –Fractal. We also apply this way and TBB (the cloud Top Black Body temperature) time series data in the weather satellite remote sensing for 1 h interval to analyze "98.7" rain-storm's nonlinear characteristic in Hubei.
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2. Wavelet –Fractal Principle and Method

2.1 time series

So-called time series [2], it is the data stream X_t that is recorded at a very particular moment by using the experiment equipments, and also it can be seen as called signals, includes the Chaos Signal [3]. To analyze Chaos Signal belongs to nonlinear time series dealing method. Actually, it needs to analyze the aspects, such as data sets up the mold, predicts, the system recognizes etc..

The satellite remote sensing affairs can be divided into the static state affairs and the dynamic state affairs, it is a dynamic state affairs of remote sensing that satellite remote sensing measures in atmosphere process. Therefore, the weather satellite TBB data is the Chaos Signal of nonlinear time series [2]. We use nonlinear time series dealing method to analyze big rain-storm nonlinear characteristic regulation at Wuhan and YC in 1998. Thus we can forecast the occurrence of the disaster weather.

2.2 Fractal

Since not everything has a certain appearance and well-regulated shape, for example, weather satellite data, length of coast line etc, we can't use traditional methods to measure, so we introduce Fractal [3].

Fractal is a parameter that is the similar property and proportion of nonlinear interaction, its calculation method is fractional dimension [4]. Generally, People study nonlinear quantity characteristic by widely adopting Von Koch fractional dimension. It defines for:

$$D=\lim_{r\to o}\frac{\ln V}{\ln \gamma} \qquad (1)$$

Fractal can be expressed by $V—\gamma^D$ with the geometry. Here, V is something estimate, γ denotes a scale of measure. At $\gamma \to 0$, it needs to have mass metrical sample to guarantee the calculating usefulness.

2.3 Wavelet analysis

Wavelet [5] is a kind of wave with special and limited length and zero average value. It has the part thin broken up function, and gets the name of "mathematics microscope". In mathematics, Wavelet is the function space L^2（R）that satisfy a function or signal ψ(x) with the following condition:

$$c_\psi = \int R^* \frac{|\psi(\omega)|^2}{|\omega|} d\omega \langle \infty \qquad (2)$$

Here, R* mean nonzero real domain. For discretionary real [a, b], among them, a must be nonzero real parameter, as following the function:

$$\psi_{(a,b)}(x) = \frac{1}{\sqrt{|a|}}\psi(\frac{x-b}{a}) \qquad (3)$$

$\psi(x)$ is called mother wavelet. Here, $\psi_{(a,b)}(x)$ is consecutive wavelet function from which mother wavelet gives birth and depends on parameter [a, b], then it is shortened as wavelet.

Wavelet comes true the signal decomposition passed a transformation [5]. The decomposed signal x (t) makes out dot product with the different α dimensions:

$$WT_x(\alpha,\tau) = \frac{1}{\sqrt{a}}\int_{-\infty}^{+\infty} x(t)\tilde{\psi}(\frac{t-\tau}{a})dt, a\rangle 0 \qquad (4)$$

The equivalent frequency domain mean for:

$$WT_x(\alpha,\tau) = \frac{\sqrt{a}}{2\pi}\int_{-\infty}^{+\infty} X(\omega)\tilde{\Psi}(a\omega)e^{+j\omega\omega}d\omega \qquad (5)$$

In the (4) and (5), $X(\omega)$ and $\tilde{\Psi}(\omega)$ is Fourier Transform with $x(t)$ and $\tilde{\psi}(t)$, but Wavelet analysis is prodigious different from Fourier analysis. Substance of Wavelet analysis is the signal decomposed in the space of a series of different layer, which can decompose the hybrid signal to constitute different frequency interweaved, and became seed signal of different frequency. They can open out internal characteristic of quantity information. Such as: Break the point, discontinuity of higher derivative, trend and self-similarity (Fractal characteristic) etc..

Based on the academic foundation of wavelet analysis, the wavelet wrap [4] can fine decompose and that fractionize not only low frequency part, but also the high frequency part. It is more elaborate disintegrated method than the wavelet analysis. Under one dimension condition, the wavelet wrap resolves to produce a two fork completed trees; Under two dimensions condition, the wavelet wrap resolves to produce a four fork completed trees, thus, the wavelet wrap analysis is a theory and technique method with more extensive application. We adopt the wavelet wrap analytical technique in

this text

3. Wavelet –Fractal vector

Fractal has better description on nonlinear characteristic, but has more requirements on enormous data samples observed, it can get the valid calculation result only on the numerous observations. And that wavelet wrap have the fine to depict the ability of subdivide. Therefore, according to the Wavelet technique and Fractal theories, we define a new physics quantity- The Vector of Wavelet –Fractal.

Assumed

$\boldsymbol{V}$ □[S1 S2 S3 …… S_k ……S_L],

here, $S_k = [x_1\ x_2\ x_3$ …… x_i …… $x_N]$, i=1,2,3,……N; k=1,2,3, …… L.

If S_k satisfy with condition of Wavelet transform, then have

$$[S_k] = [A1+D1+AA2+DA2+Ad2+DD2+ \ldots\ldots\ ]_k = [V_k]$$

Here, A1+D1+AA2+DA2+Ad2+DD2+ …… were called for wavelet decomposition signal in the multi-scale.

Then define

$$WFI=\lim_{r\to o}\frac{\ln\sum_{k=1}^{L} V_k}{\ln\gamma} \qquad (6)$$

WFI is called the Vector of Wavelet –Fractal. We can deduce the Wavelet –Fractal in the K area, it is

$$WFI_k=\lim_{\gamma_k\to 0}\frac{\ln V_k}{\ln\gamma_k} \qquad (7)$$

WFI_k is called the Vector of Wavelet –Fractal in the area.

4. An Rainstorm Instance Analysis

4.1 The nonlinear time series with the weather satellite TBB data

The nephogram of weather satellite brings forth many the atmosphere characteristics from the planet scale to the weather scale, from medium scale to small scale in various scales in the weather system. In the weather satellite, TBB data of infrared remote sensing is a kind of most modern observed data for thermodynamic process taken place in the atmosphere system. Using TBB data per hour partition in the satellite GMS-5, we can get the information on nonlinear time series, according to the TBB data in the 1998, July 1st to 31st at the Wuhan and Huangshi, we can get a nonlinear time series $\boldsymbol{X}$.

In consideration of the rain-storm process is generally 2-3 days, weather period in summer is 3-5 days. So we partition the time series X which every sections carried on by 3 days, namely S_k=[x_{k1} x_{k2} x_{k3} …… X_{k72}], and we decompose S_k sued wavelet wrap. Because the Biorthogonal wavelet is perpendicular wavelet that have tight of time-domain, it has good calculation; smooth of time-domain is bad so that it is not strictly dividing to the time-domain of signal; and dissymmetry wave of time-domain, the filter of wavelet does not have linear displacement. Biorthogonal wavelet suits data processing to the rain-storm weather system in the weather satellite infrared remote sensing. According

to (7) we can compute the Vector of Wavelet –Fractal. We carry on the glide analysis for per hour, so that analyzing the evolvement of dynamic state of weather system. We gained the vector curve of Wavelet –Fractal from July 1st to 31st.

4.2 "98.7" rain-storm analyze with the Vector of Wavelet –Fractal

Using TBB time series ***X*** in the per hour partition at the Wuhan and Huangshi in the 1998 July 1st to 31st in the satellite GMS-5, we computed the Vector of Wavelet –Fractal of **"98.7"** big rain-storm, namely value of Wavelet –Fractal at the Wuhan and Huangshi, they are WFI=1.4558 and 1.4565 (here Wavelet is decomposition of 5 layers). Taken N=72(3 day), we get the vector diagram of Wavelet –Fractal such as figure 1 and figure 2.

Figure 1 is the Vector of Wavelet –Fractal in 1998 July 1st to 31st in Wuhan. We can see obviously that the signal value descends suddenly at the horizontal sit on the 308 (July15th-16th) and 380(18th-20th), and produces infinitesimal value, namely Vector of Wavelet –Fractal occurs abnormity. We analyzed precipitation characteristic in Wuhan in the "98.7" big rain-storm, their rainfall is 77.4 mm on 17th, 285.7 mm on 21st and 171.7 mm on 22nd , and which Vector of Wavelet –Fractal occurs abnormity can run before prediction. We consider that it has nonlinear break around 16th, and the strengthening of nonlinear interaction between 18th to 20th caused the big rain-storm.

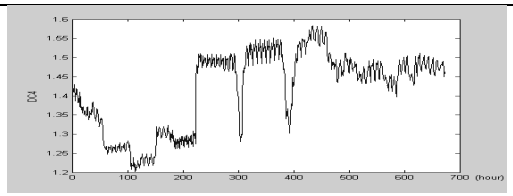

Figure 1 the Vector of Wavelet –Fractal in 1998 July 1st to 31st in Wuhan

Figure 2 is vector diagram of Wavelet-Fractal in 1998 July 1st to 31st in Huangshi. We still can see obviously it has nonlinear break signal at 18th～20th before rain-storm. But its rainfall is 139.2 mm on 21st, 360.4mm on 22nd in the big rain-storm process. It is accordant with the analytical result of Wuhan.

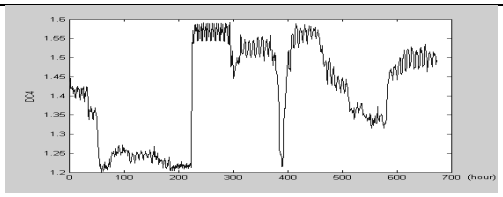

Figure 2 the Vector of Wavelet-Fractal in 1998 July 1st to 31st in Huangshi

Above-mentioned calculation results, we discover that Vector of Wavelet –Fractal occurs unconventionally before the disaster weather take place. It provides new technique and methods to predict well and accurately before occurrence of disaster weather.

4.3 Vector of Wavelet –Fractal Discussion

Figure 3 represents the Fractal curve diagram in July 1998 in Huangshi, using the same method to deal with the TBB time series data. But it only reflects a little bit opposition to undulate in the diagram, has no stronger unconventional mutation because of less calculation samples (only 72), it can't depict the details of physical process, meanwhile it can't satisfy the basic condition of V measurement, and will bring the system

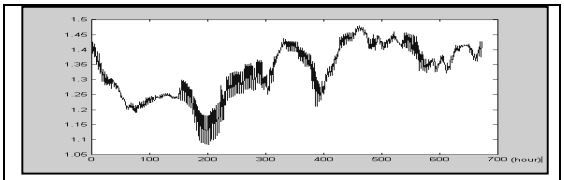

Figure 3 the Fractal in 1998 July 1st to 31st in Huangshi

indetermination error margin through the operation method. On the Vector of Wavelet –Fractal, we use to decompose only 5 layers of wavelet wrap. But its computed sample number to attain 4464, we use the wavelet wrap of "mathematics microscope", it overcomes the influence caused by artificial factor. Therefore, figure 1 and figure 2 can describe the nonlinear break, and its result are more scientific.

5. Conclusion

The Vector of Wavelet –Fractal is organic combination of wavelet analysis and Fractal theory, it is a new kind of technique method for mutual perfection. It overcomes disaster problem of dimensions brought by wavelet analysis, and the requirement of higher measure sample caused by Fractal theory. Hence the Vector of Wavelet –Fractal provides technique theories and methods for nonlinear time series analysis and applications. It is a new kind of effective technique method for the analytical satellite remote sensing information.

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